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Angular Momentum Transport between a T Tauri Star and an Accretion Disk

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Abstract.
We present results from time dependent numerical simulations of the hydromagnetic interaction between a rotating T Tauri star and a magnetically coupled Keplerian accretion disk. For a diffusive disk, we find that most of the toroidal component of the magnetic field is generated within a thin, shearing boundary layer that forms along the interface between the disk and the magnetosphere. We describe the properties of the system when it has attained a rotational equilibrium state in which the stellar spin-up produced by accretion and structural changes is compensated for by the spin-down torque arising from the magnetic connection between the star and the disk.

1. Introduction

Observations indicate that many T Tauri stars (TTS) rotate relatively slowly, despite the fact that for at least some of them, structural evolution and/or the accretion of higher specific angular momentum material from a circumstellar disk might be expected to lead to higher rates of rotation (e.g., Bouvier et al. 1986; Hartmann et al. 1986). Apparently, these stars must lose angular momentum at a rate sufficient to counteract the spin-up that would arise from evolution and accretion and remain in a state of slow rotation. An obvious candidate for producing the required angular momentum loss is the torque associated with a magnetically coupled stellar wind. However, it is difficult to account for the wide range of rotational velocities that seems to be characteristic of solar-type stars in young clusters, if wind braking is the sole mechanism for angular momentum removal during pre-main sequence evolution.

An alternative mechanism is suggested by the observations indicating that on average, TTS with accompanying disks rotate more slowly than those without (Bouvier et al. 1993; Edwards et al. 1993). For stars of the former type, a poloidal magnetic field B_p that pervades the system can furnish the means for efficiently transferring angular momentum from the star to the disk. In such a system, the field lines that connect the star to the portion of the disk outside the co-rotation radius are sheared to produce a toroidal component B_ϕ. At the surface of the star, the associated magnetic stress ($\propto B_pB_\phi$) contributes to a torque that acts to brake the stellar rotation.

In most models for the rotational evolution of TTS with disks, the magnitude of the spin-down torque is estimated without explicitly treating the dynamics of the magnetic interaction between the star and the disk. Instead, it is generally assumed that the rate at which B_ϕ is generated by differential rotation in the disk equals the the rate at which it is dissipated by turbulent diffusion, buoyancy, or reconnection. Using simplified expressions to represent these processes, estimates for both B_ϕ and the torque can then be derived (e.g., Ghosh & Lamb 1979; Cameron & Campbell 1993; Wang 1995; Ghosh 1995; Yi 1995; Armitage & Clarke 1996). In this paper, we describe preliminary results from detailed numerical simulations of a magnetically coupled star-disk system. We compare our dynamical solutions with those obtained using the 'kinematic' approach outlined above, and note some additional features not present in previous model results.

2. Model

The numerical simulations are carried out using a finite element method to solve the azimuthal components of the momentum and induction equations in spherical coordinates. We assume that the system is axisymmetric with coincident magnetic and rotational axes, and that the velocity is purely azimuthal. The prescribed poloidal magnetic field is nearly dipolar with $\nabla \times \mathbf{B}_p = 0$. B_p is furthermore taken to be independent of time, which is valid provided the conductivity of the disk is not high enough to cause significant wind-up and inflation of field. At the midplane of the disk ($\theta = \pi/2$), we require that the angular velocity Ω of the rotational motion remains Keplerian, $\Omega = \Omega_K = (GM_\star/r^3)^{1/2}$, implying that a very efficient mechanism for redistributing angular momentum operates at that location.

We treat the star as a rigidly rotating, perfect conductor, of mass $M_\star = M_\odot$, radius $R_\star = 2\,R_\odot$, and moment of inertia $I_\star = 0.2\,M_\star R_\star^2$. These values are characteristic of a fully convective, pre-main sequence star of age $\sim 10^6$ years. The magnetosphere is assumed to have a constant mass density, $\rho_m = 10^{-10}\,\mathrm{g\,cm^{-3}}$, and constant viscous and magnetic Reynolds numbers, $R_\nu = R_\eta = 10^3$, where these quantities are defined in terms of $R_\star$ and the Alfvén speed at the stellar surface. The disk occupies the region $4\,R_\star \leq r \leq 20\,R_\star$, $|\cos\theta| \leq 0.02$, and is represented by a step in the density and Reynolds numbers from their magnetospheric values to $\rho_d = 10^{-7}\,\mathrm{g\,cm^{-3}}$ and $R_\nu = R_\eta = 10$.

3. Solution Properties

We solve for the time dependent evolution of Ω and B_ϕ, starting from an initial state in which $\Omega = B_\phi = 0$ everywhere except along the disk midplane where $\Omega = \Omega_K$. At time $t = 0$, a specified, external torque is applied to the star, causing it to spin up. The increase in $\Omega_\star$ produced by this torque simulates the effects of accretion and internal structural changes on the stellar rotation rate. The simulations reveal that the system eventually attains a steady state in which the external torque acting on the star is balanced by the magnetic torque arising the star-disk interaction. Figure 1 shows the steady-state distributions of Ω and $B_\phi\, r \sin\theta$ throughout the computational domain for the solution corresponding

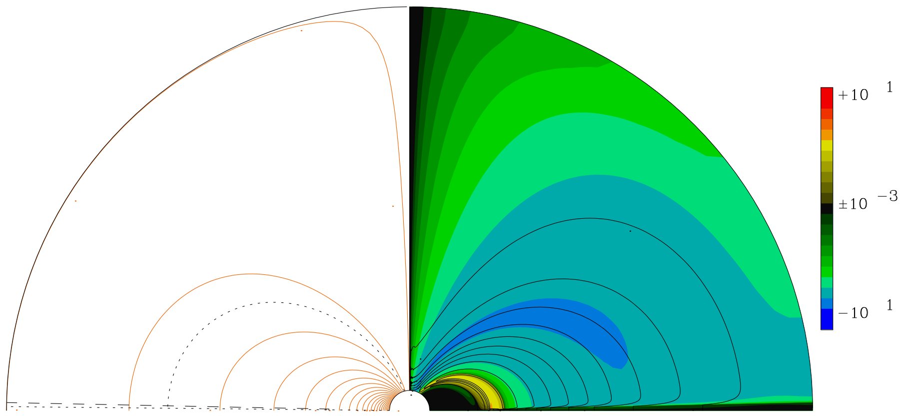

Figure 1. The left half of the figure shows the poloidal field lines (red), the boundary between the disk and the magnetosphere (dashed line), and the field line and radius along which Ω and $B_\phi\, r \sin\theta$ are plotted in subsequent figures (dotted lines). The right half of the figure shows Ω (contours) and $B_\phi\, r \sin\theta$ (bi-logarithmic color scale) for the steady-state solution discussed in §3. Within the disk, $\Omega \approx \Omega_K$ (corresponding to the boundary condition imposed at the midplane of the disk), while the field is small. Throughout most of the magnetosphere, Ω and $B_\phi r\, \sin\theta$ are constant along poloidal field lines. However, boundary layers exist just above the surface of the star and just above the surface of the disk.

to the input parameter values given in §2. In this case, $\Omega_\star \approx 6.2\, \Omega_\odot$, so that the co-rotation radius is $r_{co} = (GM_\star/\Omega_\star^2)^{1/3} \approx 5.5\, R_\star$.

Within the disk, $\Omega \approx \Omega_K$, and $B_\phi\, r \sin\theta$ increases linearly with distance s from the midplane, where s is measured along a poloidal line of force (see the right panel of Figure 2). In the magnetosphere just above the disk, there is a thin boundary layer in which the magnitudes of Ω and B_ϕ change rapidly (see the left panel of Figure 2). At a given radius r in the boundary layer, Ω assumes a value intermediate between $\Omega_K(r)$ and $\Omega_\star$, while B_ϕ increases significantly relative to its value in the disk. Some of the toroidal field generated within this shear layer diffuses in the disk where it is dissipated. Unlike most kinematic models, the rate at which B_ϕ diffuses into the disk from the boundary layer is much greater than the rate at which B_ϕ is produced inside the disk by the action of differential rotation on B_p.

Throughout most of the magnetosphere, Ω and $B_\phi\, r \sin\theta$ are constant along poloidal field lines, although the values of these quantities are not, in general, the same along adjacent lines of force. This feature is a basic property of the stationary equilibria of rotating, axisymmetric, ideal MHD systems; apparently, the magnetospheric diffusivities are low enough to permit such behavior in the present solution. The small but finite values of the viscosity and magnetic

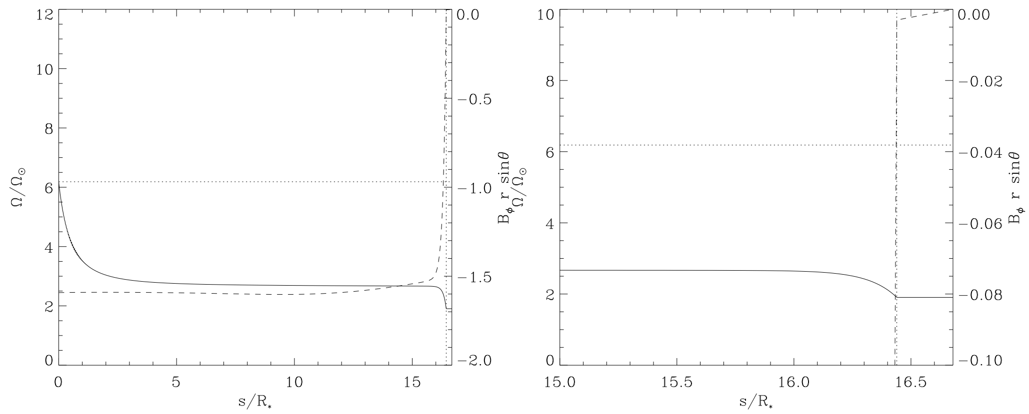

Figure 2. Angular velocity (solid line) and toroidal field (dashed line) along the poloidal field line indicated in Figure 1. The vertical dotted line marks the edge of the disk, and the horizontal dotted line shows the angular velocity of the star. The left panel shows the entire field line, while the right panel shows just the part that threads the disk and a portion of the magnetosphere directly above it. Note the boundary layers in the magnetosphere overlying both the surface of the star ($s = 0$) and the surface of the disk. Within the disk, Ω is nearly constant, while $B_\phi \, r \sin\theta$ grows approximately linearly with distance from the midplane.

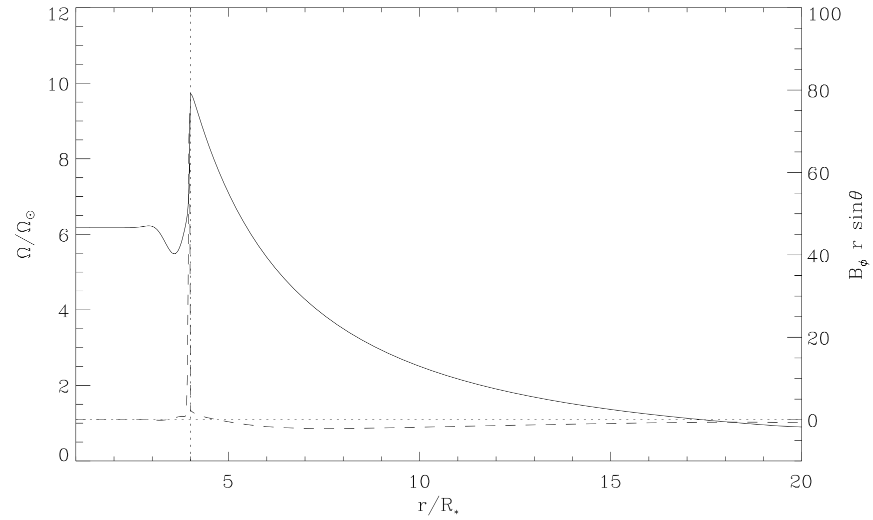

Figure 3. Angular velocity (solid line) and toroidal field (dashed line) along a radius (indicated in Figure 1) that passes through the disk. Within the disk, $\Omega \approx \Omega_K(r)$, while most of the region between the surface of the star and the inner edge of the disk rotates at $\Omega_\star$. Note the presence of a region just inside the inner edge of the disk in which the magnetospheric rotation rate is $< \Omega_\star$.

diffusivity become important in the magnetospheric layers adjacent to the stellar surface. As can be seen in Figure 2, within this region magnetic and viscous forces act to match the particular value of Ω that obtains along a given poloidal field line to the angular velocity $\Omega_\star$ of the rigidly rotating star.

In the portion of the magnetosphere that is not intercepted by poloidal field lines connecting the star and the disk, $\Omega \approx \Omega_\star$, and B_ϕ is very small (see Figure 1). However, when the corotation radius is located within the disk, there are a few field lines that intersect the equatorial plane just inside the inner edge of the disk, on which Ω is significantly lower that $\Omega_\star$ (see Figure 3). This feature can be explained as a consequence of the way in which the system of magnetospheric electrical currents is configured.

In Figure 4, it can be seen that the current density $\mathbf{J}$ $[= (c/4\pi)\,\nabla \times (B_\phi \mathbf{e}_\phi)]$ flows in two closed loops contained within meridional planes. At radii $r > r_{co}$, there is a strong cross-field current in the boundary layer that flows radially outward, and closes by way of field-aligned currents in the magnetosphere and a surface current in the (perfectly conducting) star. For $r < r_{co}$, the cross-field current in the boundary layer is directed radially inward, and again closes by way of field-aligned currents and a surface current. Inspection of Figure 4 indicates that on this latter circuit, the radial current component is non-zero at a location just interior to the inner edge of the disk. For the adopted directions of the stellar magnetic moment and angular momentum vectors, the resulting

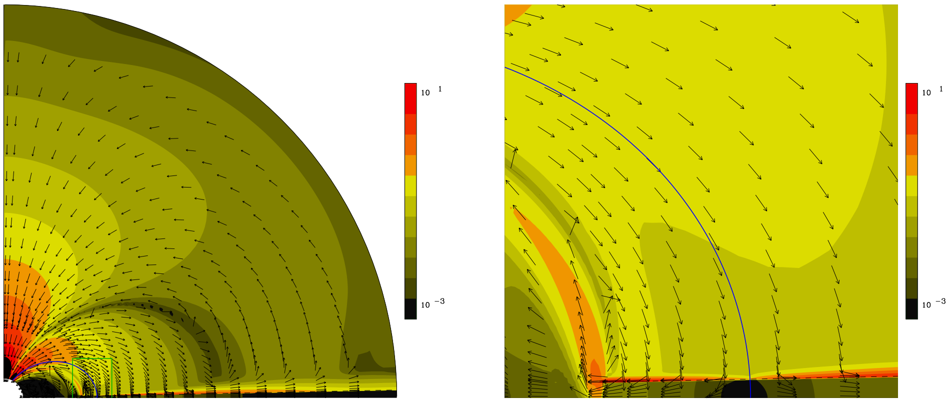

Figure 4. The electric current density, with direction indicated by arrows and magnitude by the colorscale. The blue line is the poloidal field line which intersects the equatorial plane at the co-rotation radius. In most of the magnetosphere, the current is aligned with the poloidal field lines, but in the boundary layer along the edge of the disk, there is a strong radial current. The right panel shows a region near the inner edge of the disk, indicated by the green square in the left panel. Note that just inside the inner edge of the disk there is a component of the current directed radially inward, which is necessary to close the current loop which runs along the surface of the disk. This component gives rise to a Lorentz force which causes a small region of the magnetosphere to rotate more slowly than the star.

Lorentz force, $(1/c)\,\mathbf{J} \times \mathbf{B}_p$, acts to decelerate the the rotational motion of the magnetosphere at that position.

4. Summary and Discussion

Our dynamical simulations of the magnetic interaction between a TTS and a diffusive circumstellar accretion disk yield a picture of such a system that differs in several respects from that assumed in most kinematic models. In particular, we find that the shearing boundary layer that forms at the disk surface plays a prominent role in the rotational evolution of the system, being the principal source of the toroidal field component required to transfer angular momentum from the star to the disk by magnetic means. The properties of the boundary layers that occur in our computations are known from previous MHD studies, having a structure that is analogous to, for example, the shear layer that forms around a solid body in motion through a highly conducting, magnetized fluid (e.g., Stewartson 1960a, b). Thus, analysis of our results suggests that the jumps in the magnitudes of the toroidal field and rotational velocity across the boundary layer are related according to $\Delta B_\phi \approx \sqrt{4\pi\rho_m}\,\Delta(\Omega\, r \sin\theta)$, where Δ indicates the difference between values on either side of the layer, connected by the same poloidal field line. A relation of this kind could make possible the development of a revised kinematic model, incorporating the basic properties of the present solution without the necessity of performing detailed computations.

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References

Armitage, P.J., & Clarke, C.J. 1996, MNRAS, 280, 458

Bouvier, J., Bertout, C., Benz, W., & Mayor, M. 1986, A&A, 165, 110

Bouvier, J., Cabrit, S., Fernández, M., Martín, E.L., & Matthews, J.M. 1993, A&A, 272, 176

Cameron, A.C., & Campbell, C.G. 1993, A&A, 274, 309

Edwards, S., Strom, S.E., Hartigan, P., Strom, K.M., Hillenbrand, L.A., Herbst, W., Attridge, J., Merrill, K.M., Probst, R., & Gatley, I. 1993, AJ, 106, 372

Ghosh, P., & Lamb, F.K. 1979, ApJ, 232, 259

Ghosh, P. 1995, MNRAS, 272, 763

Hartmann, L., Hewett, R., Stahler, S., & Mathieu, R.D. 1986, ApJ, 309, 275

Stewartson, K. 1960a, J. Fluid Mech., 8, 82

Stewartson, K. 1960b, Rev. Modern Phys., 32, 855

Wang, Y.-M. 1995, ApJ, 449, L154

Yi, I. 1995, ApJ, 442, 765