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Turbulent Motions with Finite Correlation Length in the Winds of K and M Supergiants

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Abstract. High resolution spectra of lines formed in the wind of evolved late-type stars suggest that the usual microturbulent assumption is not adequate to describe the nonthermal motions. We present a NLTE radiative transfer scheme that incorporates turbulent velocity fields with finite correlation length. In this approach we consider the turbulent velocity and consequently the intensity to be stochastic variables. This leads to a generalized equation of transfer having the form of a Fokker-Planck equation. In a first application we demonstate the general effects of velocity fluctuations on the line formation in expanding atmospheres.

1. Introduction

In recent years our knowledge of velocity fields in the atmospheres of late-type giants and supergiants has grown fast, mainly due to extensive UV observations with the Hubble Space Telescope (e.g. Harper 2001). Despite considerable observational and theoretical work, the mechanisms governing the mass outflow from these stars are still not well elucidated. It is conspicuous that the empirical measurements of wind parameters for individual cool stars are often controversial and do not lead to unique outflow models.

One reason for these discrepancies might be the simplistic treatment of the Doppler broadening in the microturbulent limit. It is well known that a finite velocity correlation length affects strongly the line formation. As a consequence the derived parameters as terminal wind velocity, acceleration parameter, and mass-loss rate should be interpreted with care. We have initiated a project to study the effects of stochastic velocity fields on the interpretation of wind lines in evolved late-type stars.

The theoretical principles to account for correlation effects were developed by G. Traving and collaborators (see e.g. Gail et al. 1974; Gail, Sedlmayr & Traving 1975; Traving 1975). In the framework of a first order approximation the turbulent velocity and the intensity are considered to be stochastic variables. As a consequence the classical equation of radiative transfer has to be replaced by a Fokker-Planck equation. First we present the basic principles followed by the application to a simplified wind model.

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2. Basic Formalism

In the standard scenario the expansion velocity $v(r)$ increases monotonically outward and approaches a terminal velocity v_∞ at very large distances. For the sake of simplicity we use the common β power-law description

$$v(r) = v_\infty \left(1 - \frac{R_*}{r}\right)^\beta .$$

Superimposed on this general outward motion we consider a turbulent velocity component v_t which is assumed to follow a Markov process. In this framework the turbulent motion along a line of sight is characterized by a Gaussian one-point distribution function

$$W\left(v_\mathrm{t}, s\right) = \frac{1}{\sqrt{2\pi}\sigma_\mathrm{t}} \exp\left[-\frac{v_\mathrm{t}^2}{2\sigma_\mathrm{t}^2}\right],$$

where σ_t denotes the rms turbulent velocity. The conditional probability of finding at the point $s + \Delta s$ the velocity $v_t + \Delta v_t$ is given by

$$P\left(v_\mathrm{t} + \Delta v_\mathrm{t}, s + \Delta s |\, v_\mathrm{t}, s\right) = \frac{1}{\sigma_\mathrm{t}\sqrt{2\pi\left(1-f^2\right)}} \exp\left\{-\frac{\left[\Delta v_\mathrm{t} + v_\mathrm{t}(1-f)\right]^2}{2\sigma_t^2\left(1-f^2\right)}\right\},$$

with an exponential type correlation function

$$f\left(\Delta s\right) = \exp\left[-\frac{|\Delta s|}{l}\right].$$

The correlation length l defines the length scale of the stochastic velocity variation. Alternatively the velocity v_t can be described by a corresponding Langevin equation

$$\frac{\mathrm{d}v_\mathrm{t}}{\mathrm{d}s} = -\frac{v_\mathrm{t}}{l} + \frac{\sigma_\mathrm{t}}{\sqrt{l}}\,\Gamma_{v_\mathrm{t}}(s),$$

where Γ denotes a Gaussian random variable with a zero mean and a correlation function proportional to the delta function. It can be shown that for every Langevin equation it is possible to derive a Fokker-Planck equation for the probability density of the stochastic velocity. Together with the usual radiative transfer equation

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = \kappa_\nu\left(I_\nu - S_\nu\right),$$

the differential equation for the conditional expectation value of the intensity $q_\nu(I_\nu, s|v_t, s)$ for a given velocity v_t at the point s reads

$$\frac{\partial q_\nu}{\partial s} = \frac{1}{l}\left(-v_\mathrm{t}\frac{\partial q_\nu}{\partial v_\mathrm{t}} + \sigma_\mathrm{t}^2\frac{\partial^2 q_\nu}{\partial v_\mathrm{t}^2}\right) - \kappa_\nu\left(q_\nu - S_\nu\right).$$

The outflow velocity can be incorporated defining a new variable $u = v_t + v_w$, where v_w is the projected wind velocity. This leads to the generalized radiative transfer equation

$$\frac{\partial q_\nu}{\partial s} = \frac{1}{l}\left[\left(v_\mathrm{w} - u - l\frac{\mathrm{d}v_\mathrm{w}}{\mathrm{d}s}\right)\frac{\partial q_\nu}{\partial u} + \sigma_\mathrm{t}^2\frac{\partial^2 q_\nu}{\partial u^2}\right] - \kappa_\nu\left(q_\nu - S_\nu\right).$$

The corresponding one-point distribution function changes to

$$W\left(u,s\right)=\frac{1}{\sqrt{2\pi}\sigma_{\mathrm{t}}}\exp\left[-\frac{\left(u-v_{\mathrm{w}}\right)^{2}}{2\sigma_{\mathrm{t}}^{2}}\right].$$

Finally, the expectation value of the local intensity can be obtained by a simple quadrature

$$\left\langle I_{\nu}\left(s\right)\right\rangle=\int\limits_{-\infty}^{\infty}W\left(u,s\right)q_{\nu}\left(u,s\right)\mathrm{d}u.$$

3. Application

To examine the general influence of stochastic velocity fields with finite correlation lengths on the line formation we adopt a two level pure scattering model. The opacity is assumed to vary according to the equation of continuity and is parametrized following the formalism of Baade et al. (1996):

$$\kappa(r)=\frac{v_{\infty}}{v_{\mathrm{dop}}R_{*}}\,\frac{\kappa_{0}}{(r/R_{*})^{2}\;v(r)/v_{\infty}},$$

where v_{dop} denotes the Doppler parameter containing both the thermal velocity and the turbulent contribution. The parameter κ_0 depends on the atomic quantities, the stellar radius, the mass-loss rate, and the terminal velocity. We have chosen a low-velocity outflow as observed in evolved late-type stars. The parameter β is set to the value 0.5 representing a rapid acceleration. For the sake of clarity the background spectrum is assumed to be a flat continuum. The

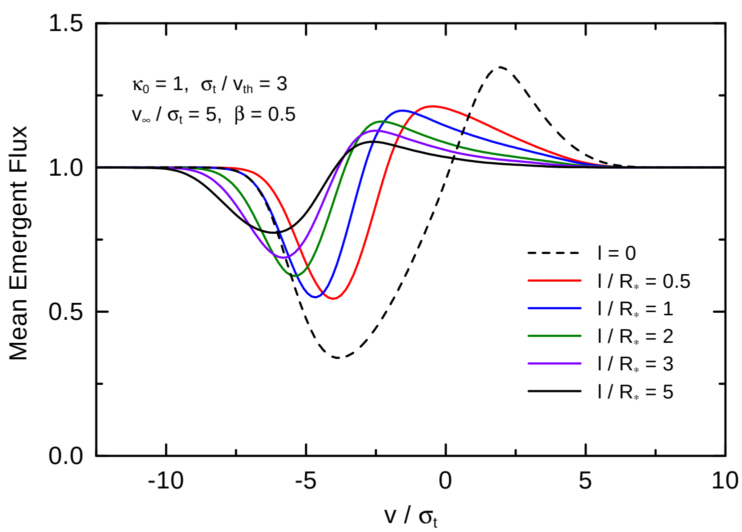

Figure 1. Mean emergent flux for different values of the correlation length in units of the stellar radius

input parameters of our calculations are only chosen to illustrate the correlation effects. We made no attempt to fit real spectra. In the Figs. 1 - 3 we present the mean emergent flux, i.e., the emergent intensity integrated over the solid angle. For comparison we show in Fig. 1 the line profile for the microturbulent limit ($l = 0$). It is obvious that with increasing correlation length the profile becomes weaker. Furthermore we find that the whole profile is shifted towards shorter wavelength. This trend can be understood by noting that with increasing correlation length large changes of the turbulent velocity get less probable. As a consequence the interaction probability between differentially moving volume elements in the wind changes dramatically.

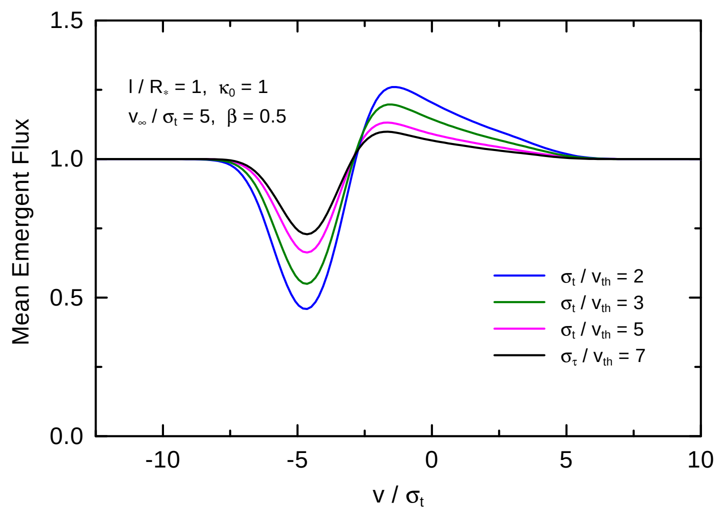

Figure 2. Mean emergent flux for different values of the ratio $\sigma_{\rm t}/v_{\rm th}$

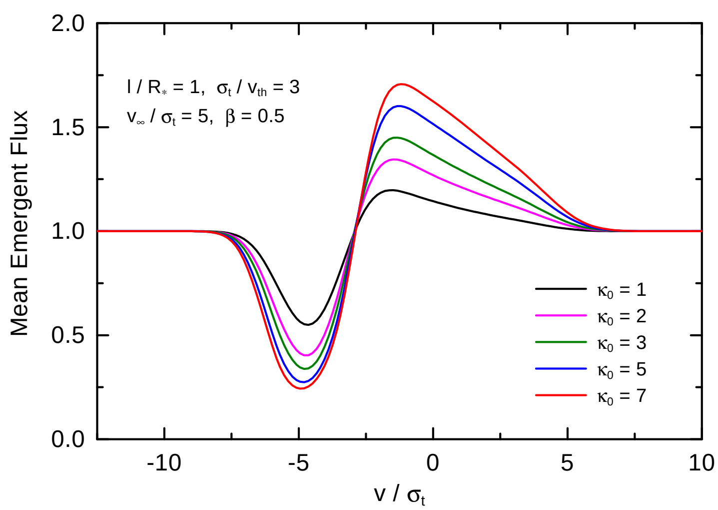

Figure 3. Mean emergent flux for different opacity parameters κ_0

4. Conclusions

We have shown that the line formation in the wind depends sensitively on the line broadening process due to stochastic large-scale motions. With a refined input model and a realistic boundary condition we will be able to match the observed spectra with theoretical profiles. In principle it should be possible to derive the stochastic parameters of the wind, since optically thin and thick lines are affected in different ways. The scale length of the turbulent motion will be an important constraint for the stochastic component of the mass outflow. These turbulent properties may play a key role to unravel the mass-loss mechanism(s) in K and M supergiants.

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