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Analytic Line Profile Diagnostics in Partial Redistribution with Continuum Absorption

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Abstract. The comoving-frame partially coherent scattering (CPCS) approximation is applied to the problem of continuum absorption during resonance-line transfer, to assess its effect on the Wilson-Bappu effect in supergiants. The basic approach is to allow the partial frequency redistribution, which normally carries photons far enough into the wings such that they escape, to be truncated by continuous absorption during the diffusion. The result is a semi-analytic conceptual approximation that reveals the mechanisms by which background opacity alters resonance-line formation and profile diagnostics.

1. The CPCS Approximation

The CPCS approximation accounts for transfer in highly coherent resonance-line wings by breaking the source function into purely coherent and completely redistributed parts. The *ansatz* for the redistribution function is such that the probability of emission at x, given absorption at x', is approximated by

$$\frac{R(x,x')}{\Phi(x')} \;=\; \Big[1 \;-\; \hat{\Gamma}(x)\Big]\,\delta(x-x') \;+\; \hat{\Gamma}(x)\hat{\Phi}(x)\;. \tag{1}$$

This redistribution function has a clear physical interpretation, where the redistributed profile $\hat{\Phi}(x)$ obeys

$$\hat{\Phi}(x) \;=\; \hat{\Gamma}(x)\Phi(x)\;, \tag{2}$$

and also note that $R(x,x')$ is properly normalized and obeys reciprocity, so it represents a physically realizable redistribution function. Its primary advantage is to decouple the wing profile into many independent monochromatic transfer problems, once the mean intensity $\hat{J}$ is specified. This may be done using first-order escape methods that are modified by the CPCS scheme itself.

A particularly important source of frequency redistribution in low-density atmospheres is Doppler diffusion, which accounts for the accumulation over many scatterings of small frequency changes ($\Delta x \sim 1$) due to atomic thermal motions. This can be accomodated within the CPCS scheme as a random walk in frequency space with $\Gamma(x) \sim 1/x^2$.

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2. The CPCS Continuum Absorption *ansatz*

Continuum opacity may be included in the CPCS scheme by adopting a heuristic *ansatz* that is motivated by results of standard one-dimensional diffusion. Since the probability of continuum absorption, as opposed to line scattering, is

$$d_\beta(x) \;=\; \frac{\beta}{\Phi(x)} \tag{3}$$

where β is the ratio of continuum to mean-line opacity, the probability of the photon reaching a given wing frequency (in Doppler units) x may be approximated as

$$P(x) \;=\; e^{-\int_0^x dx'\,\sqrt{d_\beta(x')}} \;=\; e^{-0.5x^2\sqrt{\beta/a}} \tag{4}$$

where πa is the standard Voigt parameter. Then continuum absorption is handled simply by multiplying the photon escape probability by this truncating factor in the wing, and reasonably good agreement with numerical calculations is obtained. The resulting scaling, given in Table 1, agrees with more rigorous analyses in the literature. A physical explanation for these results is given in Figure 1.

Table 1. Depth and Frequency Scaling

Continuum Opacity is	Thermalization Depth scales as:	Peak Frequency scales as:	Beyond Peaks Profile is:
Negligible	n_e^{-1}	$n_e^{-1/3}$	Gradually Falling
Dominant	$\left(\frac{\kappa_c}{\kappa_l}\right)^{-3/4}$	$\left(\frac{\kappa_c}{\kappa_l}\right)^{-1/4}$	Steeply Truncated

3. Significance for the Wilson-Bappu Effect

The Wilson-Bappu effect is a surprisingly consistent correlation between the absolute visual magnitude of a cool star and the FWHM of resonance lines formed in its chromosphere, such as the k and K lines of Mg II and Ca II, and H Lyα. Efforts to explain this effect have considered varying broadening speed, chromospheric thickness, free-electron density, and sphericity effects in extended envelopes, yet the effect remains somewhat mysterious. Here I add yet another modification to the observed trend that any complete explanation should be mindful of, since it should induce a change in the slope of the correlation in low-density giant and supergiant atmospheres: continuum absorption of wing photons.

Of the above list of effects, the one most essential to include is probably the reduction in free-electron density as the gravity falls in higher-luminosity stars. Reduced electron density implies a deeper thermalization depth and a broader profile. However, the broader a profile becomes, the greater is the importance of continuum absorption, and at some point this process should control the profile width. Perhaps the best indication that this is occurring is when the profile falls quite steeply outside its peaks (see Figure 1). Understanding how this affects the slope of the Wilson-Bappu effect requires extensive knowledge of the background opacity source at the wavelengths of each of these resonance

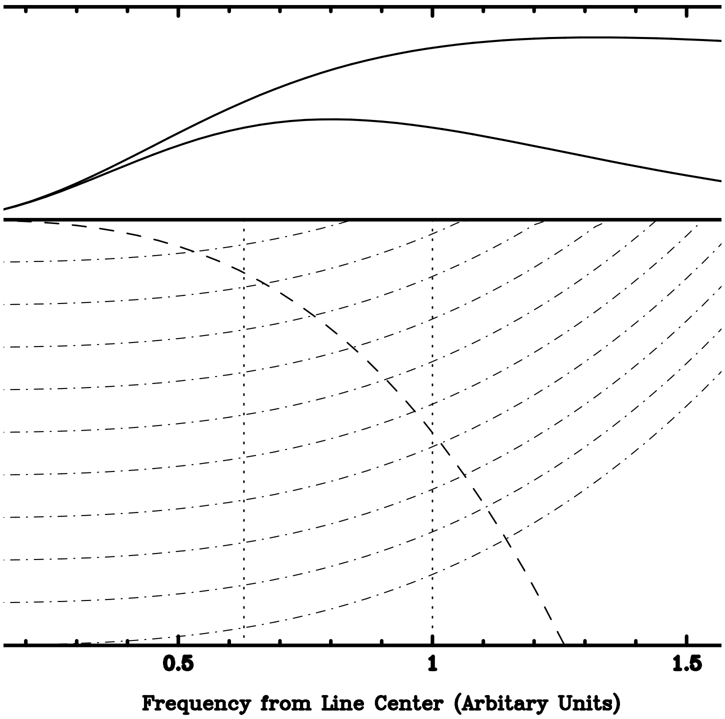

Figure 1. This graphic explains the shape of the emergent flux profile of a resonance line in an isothermal plane-parallel atmosphere with and without continuum opacity, where the line is formed by collisional creations of a core photon followed by Doppler diffusion into the wings. The dot-dashed curves at the bottom depict the average progress of photons as they simultaneously diffuse outward in both space and frequency, such that in real space the mean free path is unit monochromatic optical depth, $\Delta\tau_x \sim 1$, while in frequency space it is a unit thermal Doppler shift, $\Delta x \sim 1$. The dashed curve divides the region of primarily frequency-like diffusion on the left from primarily spacelike diffusion on the right. Note how the increasing physical mean free path implies that photons created in a given optical depth range $\Delta\tau$ are concentrated in frequency as they move into the wing, causing the profile to rise toward its peak. The emergent profiles are depicted as solid curves, the brighter of which has no continuum absorption, and the dimmer has enough continuum opacity to truncate photon escape at 1/3 the depth of the former. These profiles do not continue to rise into the wing because of collisional destruction in the former and continuum absorption in the latter. The vertical dotted lines indicate the maximum wing frequency attainable by photons before these quenching mechanisms set in.

lines, since the expected dependence on metallicity and electron density may vary substantially.

4. Conclusions

The CPCS approximation affords a simple and conceptually helpful way to obtain semi-quantitative resonance-line profiles from chromospheres under idealized conditions. The key conclusion here is that when continuum opacity is included, it may substantially narrow the profile between peaks, relative to when photon destruction is entirely collisional. Furthermore, when continuum sources of wing photons are negligible, the wings beyond the peaks are steeply truncated. An outstanding question is whether or not the chromospheres of giants and supergiants achieve low enough densities and wide enough profiles such that continuum absorption may alter the slope of the Wilson-Bappu effect. If such an alteration does occur, it is clear that the more important is the continuum, the narrower is the FWHM, and the less relevant is the column depth of the chromosphere, since the profile minimum is then determined by continuum effects and not the presence of a temperature minimum. Future work will aim at considering each of the relevant sources of continuum opacity and how they are likely to scale with chromospheric parameters such as the all-important electron density, as well as metallicity and effective temperature.