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Structure and Evolution of Low-mass Stars: Where Do Magnetic stars Become Completely Convective?

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Abstract. Stars on the main sequence are expected to be completely convective if their mass lies below a certain value, M_{cc}. Standard stellar structure codes suggest that M_{cc} is in the range (0.3-0.4)$M_{\odot}$. However, certain physical effects that are not incorporated in standard models may alter the value of M_{cc} significantly. Here we quantify the alterations that are brought about in M_{cc} when we include magnetic field effects. In particular, we modify the criterion for convective stability in the manner prescribed by Gough and Tayler (1966). We find that magnetic M dwarfs tend to have radii that are larger than expected for their T_{eff} values, or T_{eff} values that are too low for their radii. Available observational data provide quantitative support for these structural findings. Moreover, we find that, given the magnetic fields which are allowed to exist stably in low-mass stars, M_{cc} may fall to values that are as small as 0.1$M_{\odot}$. We suggest that this result is pertinent to understanding why coronae and chromospheres in active M dwarfs fail to exhibit detectable alterations at spectral class M3-M4.

1. Introduction

Stars on the lower main sequence are completely convective provided that their mass falls below a critical value M_{cc}. The question that interests us here is: what is the value of M_{cc}, and what physical factors determine this value? Standard stellar models that have been computed by a variety of investigators over the past 40 plus years suggest M_{cc} = (0.3-0.4)$M_{\odot}$.

In standard models, the value of M_{cc} is determined by the choice of opacity, equation of state, the outer boundary condition, and the parameters that enter into the convection treatment. The code we use in the present work, when it is run in standard configuration, yields $M_{cc} = 0.38M_{\odot}$, entirely consistent with previously published estimates.

Here, we consider how the effects of magnetic fields alter the value of M_{cc}.

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2. Magnetic Fields: General

In the presence of a magnetic field, ionized gas is impeded from moving across the field lines. Since turbulent convection involves in general 3-dimensional motion of gas, at least one of the components of the motion will be impeded in the presence of a field. For example, in the presence of a vertical field, horizontal motions are seriously impeded. It is true that finite electrical conductivity allows for some transverse motion: during the finite interval of time that is required for the gases to circulate in a turbulent eddy (or in a convection cell), horizontal motions of a well-defined (but reduced) magnitude can occur (see e.g. Mullan 1974). As a result, even in a strong vertical field, such as occurs in the umbra of a large sunspot, convection is not entirely suppressed.

Nevertheless, it is in general true that the presence of a vertical magnetic field makes it more difficult for convection to occur. The aim of the present work is to quantify this statement in the context of stellar structure.

3. Stellar Models Incorporating Magneto-convection

To model the interaction between magnetism and convection, we modify the usual criterion for convective stability. The Schwarzschild criterion $\nabla_{rad} > \nabla_{ad}$ is both necessary and sufficient to ensure convective instability in hydrodynamic flow. (Here, $\nabla = d\log T/d\log p$ is the temperature gradient relative to the pressure.)

However, in a magnetohydrodynamic situation, it is no longer possible to obtain unique conditions that are both necessary and sufficient for instability. The simplest approach to deriving a relevant criterion in the presence of a vertical field B has been described by Gough and Tayler (1966: hereafter GT). GT obtained the following condition as one that is sufficient to ensure convective stability:

$$\nabla_{rad} < \nabla_{ad} + \delta \qquad (GT)$$

where δ is roughly equal to $B^2/4\pi\gamma P_{gas}$. A more complicated approach to deriving conditions for instability can be found in Lydon and Sofia (1995), but for simplicity, we confine ourselves to GT in this exploratory study.

The GT criterion indicates that in the presence of a vertical field, ∇_{rad} may exceed ∇_{ad} by a finite amount without having convection set in. This is a quantitative statement of the fact that magnetic fields make it harder for convection to occur.

Even when convection does set in, the presence of the magnetic field reduces the efficiency with which energy transport can occur (GT). As a result, the convective solution for a magnetic envelope lies on a different adiabat from the one that would be obtained in the non-magnetic analog. Because of the lower value of entropy in the magnetic envelope, a magnetic star has a different internal structure. We will quantify this statement below.

4. An Approach to Modelling the Internal Magnetic Field Structure

We use a code that has been used previously to compute stellar evolution, but with the Schwarzschild criterion replaced with eqn. (GT). The principal source of uncertainty in the present work is in the choice of a radial profile for the parameter δ. The simplest assumption is to assume $\delta(r) = \text{const}$ (as assumed in a model developed for G stars by Ventura et al. 1998).

It may be more realistic to allow $\delta(r)$ to increase with increasing radial distance from the center of the star. Observations of the magnetic field strengths on the surfaces of active stars indicate that the fields may be nearly in equipartition with the photospheric pressure (Saar 1996). This means that the numerical value of δ is close to unity (within a factor of 2 or so) in the surface layers. It is hard to imagine how δ could become larger than this inside the star. More likely, the interior values of δ are smaller than the values at the surface. We have computed a set of models in which $\delta(r)$ is chosen to have the functional form $\sim [m(r)/M_*]^{2/3}$.

There is a second magnetic effect that we incorporate into some of our models: once we decide on a radial profile of δ, the term dp_{mag}/dr is included in the equation of hydrostatic equilibrium.

5. Expanded Stars, Radiative Cores

In our code, we evolve stars of various masses (0.1-$0.6 M_\odot$) from the Hayashi track to ages of 5 GYr. The first important result which emerges from our work is: *for two stars with the same mass, the magnetic star has a larger radius and a smaller T_{eff} than the non-magnetic star.*

Observational data support these model results. Leggett et al. (2000) have used infra-red spectrophotometry to obtain accurate estimates of R_* and T_{eff} for a sample of red dwarfs. Independent determinations of R_* and T_{eff} are also available from eclipsing binaries (see Mullan and MacDonald 2001). When these data are plotted in a T_{eff} versus R_* diagram (see Fig. 1), active stars and inactive stars can be distinguished in the following sense: active stars are displaced (relative to inactive stars) towards larger radii and smaller T_{eff}. This displacement is precisely what our models predict for magnetic stars *vis-a-vis* non-magnetic stars of the same mass.

To quantify the distinct populations of active (magnetic) and inactive (non-magnetic) stars, we measured the perpendicular distance Δ of each star in the T_{eff} versus R_* diagram from a reference curve (see Fig. 1). (The reference curve we chose was the Baraffe-Chabrier [1997] curve with solar metal abundances.) We assign a positive (negative) value to Δ if the point lies below (above) the reference curve. We normalize the value of Δ for each point by taking the ratio of Δ to the total error bar associated with that point. The cumulative distributions of Δ values for active and inactive stars are used for a Kolmogoroff-Smirnov test as shown in Fig. 2. Given the sizes of the two samples, the value of D_{max} shown in Fig. 2 indicates that active and inactive stars represent distinct populations with a confidence level in excess of 99.9%.

Our second important result concerns *the presence of a radiative core in low mass stars.* We find that by choosing δ to have values of 0.06 or less, convection

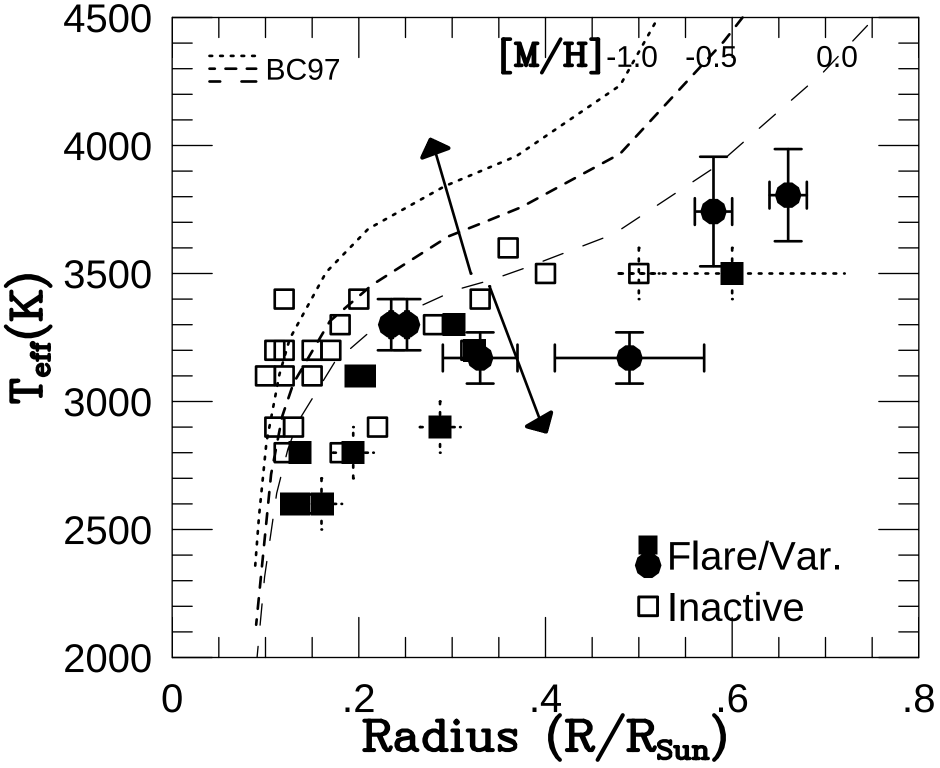

Figure 1. M dwarfs in the T_{eff}-radius diagram. Filled symbols: active stars. Open symbols: inactive stars. The parameter Δ has positive (negative) values along the downward (upward) arrow.

is suppressed in the core of stars with masses as low as $0.1M_{\odot}$. Smaller values of δ suffice to suppress convection in the core of stars with larger masses (up to $0.38M_{\odot}$).

Thus, even a star with mass $0.1M_{\odot}$ (i.e. spectral type M7-M8) may have a radiative core provided that the magnetic field strength in the star exceeds a certain value. The field that suffices to suppress convection in a star of mass $0.1M_{\odot}$ is about 100 MG.

The field strength that is *necessary* to create a radiative core in a $0.1M_{\odot}$ star is certainly less than the above estimate.

Are fields of 10's of MG stable in low mass stars? To answer this, we refer to work by Schussler et al. (1996), where there is a study of non-axisymmetric instabilities in a $1M_{\odot}$ star of various ages and various rotational periods. Referring to their case in which the star has a deep convective envelope, we find that

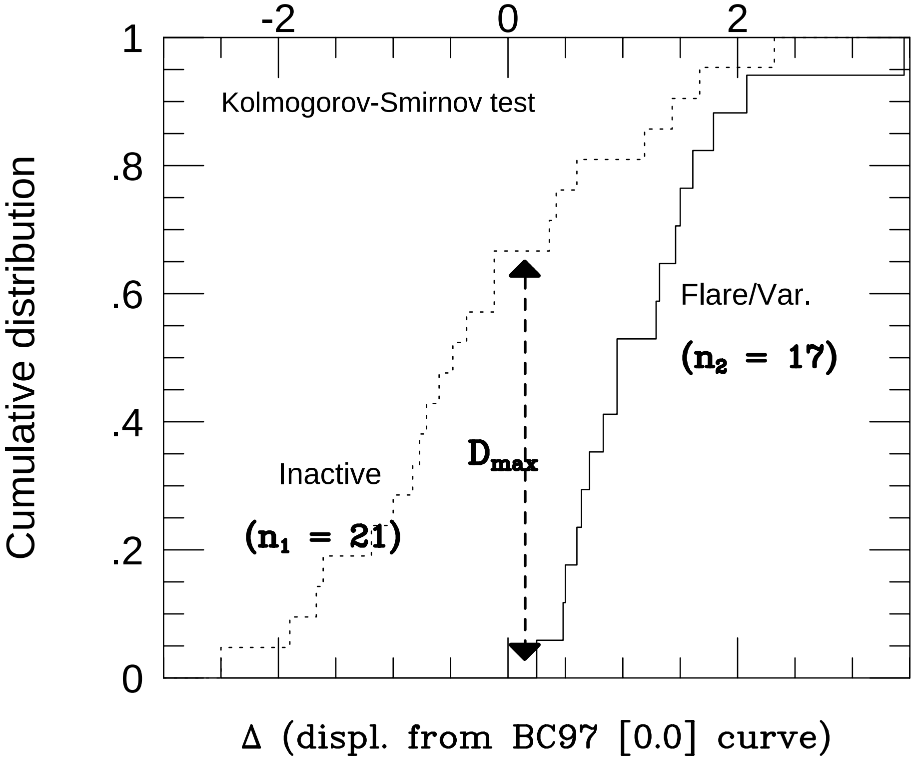

Figure 2. Kolmogoroff-Smirnov test to demonstrate distinct populations of active and inactive stars.

fields of up to 100 MG are indeed stable in stars that rotate as fast as some of the low-mass active stars.

Therefore, by assuming the presence of fields of allowable strength in low mass stars, we suggest that the onset of complete convection on the lower main sequence may be as late as M7-M8. This is considerably later in type than that predicted by standard models (M3-M4).

6. Transitions in Coronal and Chromospheric Emission

Our result that M7-M8 is the spectral type for the onset of complete convection is of interest as regards certain observed properties of the coronae and chromospheres in active cool dwarfs. To see why this is so, we note that activity is generally associated with dynamo operation. Now, in the case of the Sun and solar-like stars, a significant component of the dynamo is associated with the interface between radiative core and convective envelope. In a completely convective star, such an interface dynamo cannot operate, and the dynamo must operate in a non-solar mode.

The theoretical result $M_{cc} = (0.3\text{-}0.4)M_\odot$, with its predicted disappearance of interface dynamos at M3-M4, led to the expectation that a transition might occur in the activity properties of red dwarfs at these spectral types. As an indicator of coronal activity, we refer to L_X/L_{bol}. As an indicator of chromospheric activity, we refer to $L_{H\alpha}/L_{bol}$.

A long-standing problem with these predictions is that no sign of a transition occurs at M3-M4 in either of these activity indicators (Fleming et al. 1993; Hawley et al. 1999).

Instead, in order to have a significant alteration in the values of L_X/L_{bol} or $L_{H\alpha}/L_{bol}$, the data suggest that one must go to spectral types as late as M7-M8.

We note that these spectral types correspond to the location where radiative cores are expected to disappear in magnetic stars.

We therefore propose that the dramatic decreases in coronal and chromospheric activity levels in M dwarfs later than M7-M8 occur because the interface dynamo has disappeared in these stars.

A more detailed account of this work can be found in Mullan and MacDonald (2001).

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