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Internal Dynamics of Slowly Rotating Stars

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Abstract.

In slowly rotating stars, the coupling of helium settling and meridional circulation leads to a particular hydrodynamical process which was not introduced in previous computations of abundance variations. The μ-gradients induced by element settling can lead to a quasi-paralysis stage in which the efficiency of both the circulation and the microscopic diffusion are strongly reduced. Such an effect was studied in the eighties by Mestel et al. but in the stellar cores where the μ-gradients can lead to a process which they called "creeping paralysis". Below convective zones, this process leads to another kind of mixing which may have important consequences. We show that this effect can account for the lithium plateau in halo stars and discuss the solar case.

1. Diffusion and Mixing in the Presence of μ-Gradients

The meridional circulation velocity in stars, in the presence of μ-gradients, is the sum of two terms, one due to the classical thermal imbalance (Ω-currents) and the other one due to the induced horizontal μ-gradients (μ-induced currents, or μ-currents in short). In the most general cases, μ-currents are opposite to Ω-currents (Mestel 1953, Zahn 1992, Maeder and Zahn 1998). When element settling occurs below the stellar outer convective zone in cool stars, a small helium gradient builds, even in the presence of circulation. Vauclair (1999) (paper I) showed that, in slowly rotating stars, the resulting μ-gradients are rapidly large enough to create μ-currents of the same order as Ω-currents. Then a kind of "creeping paralysis" can settle below the convective zone, in the same way as discussed by Mestel (1953) and Mestel and Moss (1986) for nuclearly-induced μ-gradients.

Here we have computed these currents in halo stars for different metallicities and rotation velocities and we have done a complete treatment of the resulting diffusion of the chemical species, including mixing and settling. We also did preliminary computations for the solar case.

The vertical meridional circulation velocity may be written (paper I) :

$$u_r = \frac{\nabla_{\rm ad}}{\nabla_{\rm ad} - \nabla + \nabla_\mu} \, \frac{\varepsilon_\Omega}{g} \tag{1}$$

where g represents the local gravity, $\nabla_{\rm ad}$ and ∇ the usual adiabatic and real ratios $\left(\dfrac{d\ln T}{d\ln P}\right)$ and ∇_μ the mean molecular weight contribution $\left(\dfrac{d\ln \mu}{d\ln P}\right)$.

The expression of ε_Ω is obtained as a function of the Ω and μ-currents :

$$\varepsilon_\Omega = \left(\frac{L}{M}\right)(E_\Omega + E_\mu)\, P_2(\cos\theta) \tag{2}$$

with:

$$E_\Omega = \frac{8}{3}\left(\frac{\Omega^2 r^3}{GM}\right)\left(1 - \frac{\Omega^2}{2\pi G\overline{\rho}}\right) \tag{3}$$

$$E_\mu = \frac{\rho_m}{\overline{\rho}}\left\{\frac{r}{3}\,\frac{d}{dr}\left[\left(H_T\frac{d\Lambda}{dr}\right) - (\chi_\mu + \chi_T + 1)\Lambda\right] - \frac{2H_T\Lambda}{r}\right\} \tag{4}$$

Here $\overline{\rho}$ represents the density average on the level surface ($\simeq \rho$) while ρ_m is the mean density inside the sphere of radius r; H_T is the temperature scale height; Λ represents the horizontal μ fluctuations $\frac{\tilde{\mu}}{\overline{\mu}}$; χ_μ and χ_T represent the derivatives :

$$\chi_\mu = \left(\frac{\partial \ln \chi}{\partial \ln \mu}\right)_{P,T} \quad ; \quad \chi_T = \left(\frac{\partial \ln \chi}{\partial \ln T}\right)_{P,\mu} \tag{5}$$

Λ refers to the horizontal μ gradients which result from the coupling of the vertical μ gradients and the circulation.

The vertical μ-gradients are induced by element settling, which is computed as in Richard et al. (1996).

2. The Results for Halo Stars

Figure 1 presents the variations of $|E_\mu|$ with time in a $0.75M_\odot$ halo star. At the beginning of the stellar evolution, $|E_\mu|$ is smaller than $|E_\Omega|$; meridional circulation proceeds and induces transport of chemical elements coupled with settling. The helium concentration gradient below the convective zone increases (in absolute value) leading to an increasing vertical μ-gradient . The horizontal μ-gradient Λ follows proportionally until $|E_\mu|$ becomes of the same order as $|E_\Omega|$. Then we assume that the μ-gradient in this zone remains of the order of the critical value for which the circulation vanishes (see Théado and Vauclair 2001 for details).

In this case, computations of lithium diffusion in halo stars lead to results close to the “lithium plateau”, with a very small dispersion and an overall depletion by a factor of about two. The resulting primordial lithium is 2.5±0.1, consistent with D/H and ^{4}He/H (Burles and Tytler 1998). These results give a baryonic number $\eta = (5 \pm 1).10^{-10}$ and a baryonic density $\Omega_b h^2$ between 0.015 and 0.022.

3. The Solar Case

Similar computations have been applied to the solar case, in a preliminary way. In previous work, it was inferred that the rotation-induced mixing became inefficient as soon as the μ gradient became larger than some critical value (Richard

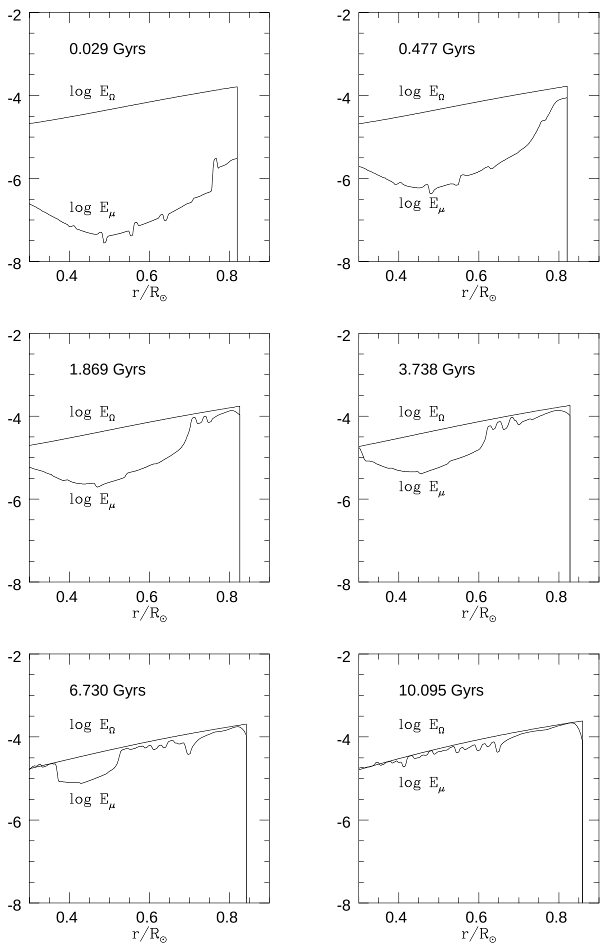

Figure 1. Evolution of μ-currents with time inside the star. The graphs show the variations with depth of both $|E_\Omega|$ and $|E_\mu|$ (defined in eq. 3 and 4) in a $0.75M_\odot$ halo star with $[Fe/H] = -2$ and $V_{rot} = 5$ km.s^{-1}.

et al. 1996). Here we include the new treatment of the physics of diffusion-induced μ gradients as described above. In these preliminary computations, we assume a rotational braking following a Skumanich law and solid rotation in the radiative zone. Better computations will be done in the future with a more realistic braking law (Pinsonneault 2001). We use the OPAL96 (Iglesias and Rogers 1996) opacities complemented at low temperatures with the opacities by Alexander and Ferguson (1994). Nuclear reaction rates are computed using the NACRE reaction rates.

The new models have been tested for their sound velocities and compared with Basu (1997) seismic reults. We show that the induced mixing slightly decreases the helium gradient below the convective zone and that it may erase the peak-shaped discrepancy generally found between the observed and computed sound velocities in this region. It may also account for the observed lithium depletion.

Full results will be given in a forthcoming paper (Théado, Richard and Vauclair, 2001).

4. Conclusion

We have proved that the computations of rotation-induced mixing must include the feed-back effect of diffusion-induced μ-gradients on the meridional circulation. This effect, which was not included in previous computations, can account for the small dispersion in the lithium plateau of halo (pop II) main-sequence stars. It can also help understanding why the solar radiative zone is not mixed except in a thin layer below the convective zone.This effect will be computed for other stars and could possibly be tested in future asteroseismological experiments.

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